

A method of thermal shielding of an object with the help of a material with transverse pores and forced filtering of a gaseous coolant through the material is studied.

In the process of operating and repairing power-saturated equipment as well as in eliminating faults it is necessary to provide effective thermal shielding of surrounding objects from high temperatures and strong fluxes of infrared radiation. Service personnel must also be reliably protected from these factors. The use of standard thermal insulation often does not give the desired result.

Calculations show that the problem can be solved by using active thermal shielding which insulates objects with a thermally insulating material containing transverse pores and with part of the penetrating heat removed by gaseous coolant forced through the system. This method reduces the average temperature of the thermal insulation and preserves its structural properties [1]. The external conditions of heat exchange also change owing to the interaction of the gas filtering through the system with the boundary layer on the surface of the thermal insulator [2].

This paper is devoted to an analysis of the increase in the characteristic thermal resistance of insulation.

For simplicity we shall study the process of thermal shielding for the example of a flat, infinite plate having a thickness Δ and made of a material containing transverse pores and having a constant effective thermal conductivity λ_{eff} , which can be represented to some approximation in the form

$$\lambda_{\text{eff}} = \lambda_{\text{fr}} + \lambda_{\text{g}}$$

If T_0 and T_1 are the temperatures of the surfaces of the plate, then according to Fourier's law the density of the heat flux flowing through it is given by

$$q_{\text{eff}} = q_{\text{fw}} + q_{\text{g}} = (\lambda_{\text{fw}} + \lambda_{\text{g}}) \frac{T_1 - T_0}{\Delta} = \frac{T_1 - T_0}{R_{\text{g}}^0}. \quad (1)$$

We shall now direct gas with a specific heat capacity c_p and flux density j through a plate on the side of the object being protected, whose temperature is T_0 . Then as a result of heat transfer from the material of the plate to the gas filtering through the system the heat flux in an infinitely thin layer of the plate with unit area is equal to

$$dq_{\alpha} = \alpha_v (T_{\text{fw}} - T_{\text{g}}) dx. \quad (2)$$

The heat flux spreading along the framework of the plate plays the role of a heat sink dq_{fw} , which is related with the temperature of the framework T_{fw} by the expression

$$dq_{\text{fw}} = \lambda_{\text{fw}} \frac{d^2 T_{\text{fw}}}{dx^2} dx. \quad (3)$$

A similar relation holds between the heat flow out of the gas phase dq_{g} and its temperature T_{g} :

$$dq_{\text{g}} = \lambda_{\text{g}} \frac{d^2 T_{\text{g}}}{dx^2} dx. \quad (4)$$

In accordance with the energy balance both thermal flows are transmitted to the filtering gas and increase its enthalpy:

$$dq_{fw} + dq_g = dh = jc_p dT_g. \quad (5)$$

Thus we can write a system of two differential equations, describing the distribution of the temperature of the framework of the plate and the gas, with two unknowns:

$$\lambda_{fw} \frac{d^2 T_{fw}}{dx^2} = \alpha_v (T_{fw} - T_g), \quad \lambda_{fw} \frac{d^2 T_{fw}}{dx^2} + \lambda_g \frac{d^2 T_g}{dx^2} = jc_p \frac{dT_g}{dx}. \quad (6)$$

The system (6) reduces to a fourth-order linear differential equation with one unknown

$$-\frac{\lambda_g}{jc_p} \frac{d^4 T_{fw}}{dx^4} + \frac{d^3 T_{fw}}{dx^3} + \frac{\alpha_v}{jc_p} \left(1 + \frac{\lambda_s}{\lambda_{fw}}\right) \frac{d^2 T_{fw}}{dx^2} - \frac{\alpha_v dT_{fw}}{\lambda_{fw} dx} = 0. \quad (7)$$

The general solution of Eq. (7) has the form

$$T_{fw}(x) = C_1 \exp(\xi_1 x) + C_2 \exp(\xi_2 x) + C_3 \exp(\xi_3 x) + C_4, \quad (8)$$

where ξ_1 , ξ_2 , and ξ_3 are the roots of the characteristic cubic equation

$$-\frac{\lambda_s}{jc_p} \xi^3 + \xi^2 + \frac{\alpha_v}{jc_p} \left(1 + \frac{\lambda_T}{\lambda_{fw}}\right) \xi - \frac{\alpha_v}{\lambda_{fw}} = 0.$$

Once the analytical form of the roots has been determined [3] it is not difficult to find, using the boundary conditions on the surfaces of the plate, the values of the integration constants and to obtain the specific form of the general solution [8].

For analysis its simpler to solve the problem when the effective thermal conductivity of the plate is determined by the thermal conductivity of its framework, i.e., $\lambda_{eff} \approx \lambda_{fw} \gg \lambda_g$. (Similar assumptions were also made in [4, 5].)

In this case the starting system of equations can be written as follows:

$$\lambda_{fw} \frac{d^2 T_{fw}}{dx^2} = \alpha_v (T_{fw} - T_g), \quad \lambda_{fw} \frac{d^2 T_{fw}}{dx^2} = jc_p \frac{dT_g}{dx}. \quad (6')$$

The system (6') reduces to one equation

$$\frac{jc_p}{\alpha_v} \frac{d^3 T_{fw}}{dx^3} + \frac{d^2 T_{fw}}{dx^2} - \frac{jc_p}{\lambda_{fw}} \frac{dT_{fw}}{dx} = 0, \quad (7')$$

whose general solution has the form

$$T_{fw}(x) = C_1 \exp(\psi_1 x) + C_2 \exp(\psi_2 x) + C_3. \quad (8')$$

The roots ψ_1 and ψ_2 of the characteristic equation

$$\frac{jc_p}{\alpha_v} \psi^2 + \psi - \frac{jc_p}{\lambda_{fw}} = 0$$

can be expressed as

$$\psi_{1,2} = -\frac{\alpha_v}{2jc_p} \pm \sqrt{\left(\frac{\alpha_v}{2jc_p}\right)^2 + \frac{\alpha_v}{\lambda_{fw}}}. \quad (9)$$

If the temperature of the filtering as at the inlet to the plate is equal to the temperature of the surface of the plate, then the boundary conditions of the problem will be as follows:

$$T_{fw}|_{x=0} = T_0; \quad T_{fw}|_{x=\Delta} = T_1; \quad \left. \frac{d^2 T_{fw}}{dx^2} \right|_{x=0} = 0.$$

The particular solution satisfying these conditions will be

$$T_{fw}(x) = T_0 + (T_1 - T_0) \frac{[\exp(\psi_1 x) - 1] - \left(\frac{\psi_1}{\psi_2}\right)^2 [\exp(\psi_2 x) - 1]}{[\exp(\psi_1 \Delta) - 1] - \left(\frac{\psi_1}{\psi_2}\right)^2 [\exp(\psi_2 \Delta) - 1]}. \quad (10)$$

The temperature of the gas can be found by using the first equation of the system (6'). It follows from it, using (10), that

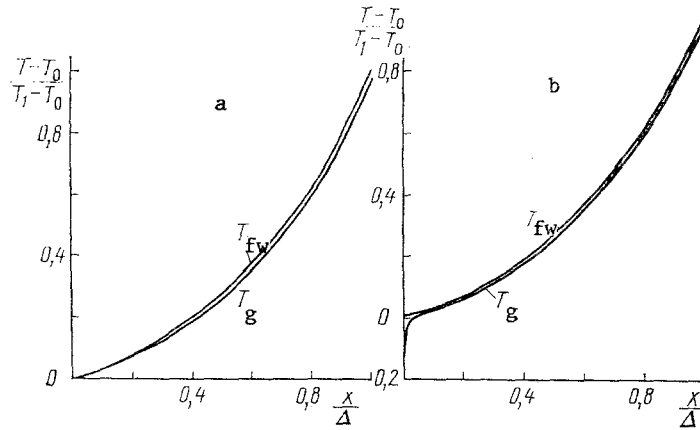


Fig. 1. The distribution of the temperature of the frame-work of a porous plate T_{fw} and the filtering gas T_g with $T_g|_{x=0}$ (a) and with $dT_{fw}/dx|_{x=0} = 0$ (b). The temperature curves were constructed for the case $\Delta = 4 \cdot 10^{-2}$ m, $\lambda_{fw} = 10^{-1}$ BT/(m·K), $j = 5 \cdot 10^{-2}$ Kg/(m²·c), $c_p = 10^3$ / (KT·K), $\alpha_v = 10^4$ BT/(M³·K)

$$\begin{aligned}
 T_g(x) &= T_{fw}(x) - \frac{\lambda_{fw}}{\alpha_v} \frac{d^2 T_{fw}}{dx^2} = \\
 &= T_0 + (T_1 - T_0) \left\{ \frac{[\exp(\psi_1 x) - 1] - \left(\frac{\psi_1}{\psi_2}\right)^2 [\exp(\psi_2 x) - 1]}{[\exp(\psi_1 \Delta) - 1] - \left(\frac{\psi_1}{\psi_2}\right)^2 [\exp(\psi_2 \Delta) - 1]} \right. \\
 &\quad \left. - \frac{\lambda_{fw}}{\alpha_v} \psi_1^2 \frac{\exp(\psi_1 x) - \exp(\psi_2 x)}{[\exp(\psi_1 \Delta) - 1] - \left(\frac{\psi_1}{\psi_2}\right)^2 [\exp(\psi_2 \Delta) - 1]} \right\}.
 \end{aligned} \tag{11}$$

The form of the dependences (10) and (11) is presented in Fig. 1a.

The expression for the heat flux penetrating through the plate and reaching the object being protected can be obtained from the expression (10) using the dependence

$$\begin{aligned}
 q|_{x=0} &= -\lambda_{fw} \frac{dT}{dx} \Big|_{x=0} = \\
 &= \lambda_{fw} \frac{T_1 - T_0}{\Delta} \frac{\psi_1 \Delta - \left(\frac{\psi_1}{\psi_2}\right)^2 \psi_2 \Delta}{[\exp(\psi_1 \Delta) - 1] - \left(\frac{\psi_1}{\psi_2}\right)^2 [\exp(\psi_2 \Delta) - 1]} = \frac{T_1 - T_0}{R_r^*}.
 \end{aligned} \tag{12}$$

Comparing the expressions (1) and (12) makes it possible to determine the effectiveness of the active thermal shielding. If it is characterized by the coefficient R_T^*/R_T^0 , which we shall call the coefficient of active thermal shielding, then it will be determined by the following relation:

$$n = \frac{[\exp(\psi_1 \Delta) - 1] - \left(\frac{\psi_1}{\psi_2}\right)^2 [\exp(\psi_2 \Delta) - 1]}{\psi_1 \Delta - \left(\frac{\psi_1}{\psi_2}\right)^2 \psi_2 \Delta}. \tag{13}$$

If, however, complete thermal shielding of the object must be achieved, the temperature of the injected gas must be lowered relative to the temperature of the inner surface of the plate. We shall find for this the solution of Eq. (7') with the following boundary conditions:

$$T_{fw}|_{x=0} = T_0; T_{fw}|_{x=\Delta} = T_1; \left. \frac{dT_{fw}}{dx} \right|_{x=0} = 0.$$

After the integration constants in Eq. (8') are determined we obtain the following expression for the temperature of the plate framework:

$$T_{fw}(x) = T_0 + (T_1 - T_0) \frac{[\exp(\psi_1 x) - 1] - \frac{\psi_1}{\psi_2} [\exp(\psi_2 x) - 1]}{[\exp(\psi_1 \Delta) - 1] - \frac{\psi_1}{\psi_2} [\exp(\psi_2 \Delta) - 1]} \quad (14)$$

The temperature of the filtering gas will then be

$$\begin{aligned} T_g(x) &= T_{fw}(x) - \frac{\lambda}{\alpha_v} \frac{d^2 T_{fw}(x)}{dx^2} = \\ &= T_0 + (T_1 + T_0) \left\{ \frac{[\exp(\psi_1 x) - 1] - \frac{\psi_1}{\psi_2} [\exp(\psi_2 x) - 1]}{[\exp(\psi_1 \Delta) - 1] - \frac{\psi_1}{\psi_2} [\exp(\psi_2 \Delta) - 1]} - \right. \\ &\quad \left. - \frac{\lambda_{fw}}{\alpha_v} \psi_1 \frac{\psi_1 \exp(\psi_1 x) - \psi_2 \exp(\psi_2 x)}{[\exp(\psi_1 \Delta) - 1] - \frac{\psi_1}{\psi_2} [\exp(\psi_2 \Delta) - 1]} \right\}. \end{aligned} \quad (15)$$

The distribution of the temperatures $T_{fw}(x)$ and $T_g(x)$ in the plate is shown in Fig. 1b.

An even simpler case is the case when owing to intense heat exchange between the plate framework and the gas, local thermal equilibrium $T_{fw}(x) \approx T_g(x)$ holds. To analyze this case we shall transform the expression (9), multiplying and dividing it by the conjugate expression:

$$\begin{aligned} \psi_{1,2} &= -\frac{\alpha_v}{2jc_p} \pm \sqrt{\left(\frac{\alpha_v}{2jc_p}\right)^2 + \frac{\alpha_v}{\lambda_{fw}}} = \\ &= \frac{\frac{\alpha_v}{\lambda_{fw}}}{\frac{\alpha_v}{2jc_p} \pm \sqrt{\left(\frac{\alpha_v}{2jc_p}\right)^2 + \frac{\alpha_v}{\lambda_{fw}}}} = \frac{jc_p}{\lambda_{fw}} \frac{2}{1 \pm \sqrt{1 + \frac{(2jc_p)^2}{\alpha_v \lambda_{fw}}}}. \end{aligned} \quad (16)$$

It follows from here that for $(jc_p)^2 / \alpha_v \lambda_{fw} \ll 1$, i.e., for a definite combination of the quantities characterizing the forms of heat transfer within the plate, the roots of the characteristic equation will have the following form:

$$\psi_1 = \frac{jc_p}{\lambda_{fw}}; \psi_2 = -\frac{\alpha_v}{jc_p}. \quad (9')$$

The expression characterizing the maximum temperature nonuniformity in the plate, as follows from Eq. (11) (see also Fig. 1a), is equal to the relative difference of the temperatures of the framework and gas at the surface of the plate:

$$\begin{aligned} \delta T|_{x=\Delta} &= \frac{(T_{fw} - T_g)|_{x=\Delta}}{T - T_0} = \\ &= \frac{\lambda_{fw}}{\alpha_v} \psi_1^2 \frac{\exp(\psi_1 \Delta) - \exp(\psi_2 \Delta)}{[\exp(\psi_1 \Delta) - 1] - \left(\frac{\psi_1}{\psi_2}\right)^2 [\exp(\psi_2 \Delta) - 1]}. \end{aligned}$$

Analysis of this expression together with the formula (16) leads to the following conclusions: as the thickness of the plate is increased the quantity $\delta T|_{x=\Delta}$ approaches the value $(jc_p)^2 / \alpha_v \lambda_{fw}$, coming closest to it for thicknesses satisfying the condition $\Delta \gg \sqrt{\lambda_{fw} / \alpha_v}$. Thus in order for there to be a temperature equilibrium between the phases in the plate the following system of inequalities must be satisfied:

$$jc_p \ll \sqrt{\alpha_v \lambda_{fw}} \Delta \gg \sqrt{\frac{\lambda_{fw}}{\alpha_v}}; \quad (17)$$

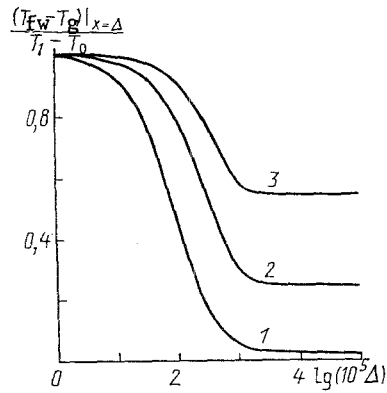


Fig. 2

Fig. 2. The relative difference of the temperatures of the framework and gas on the outer surface of the plate $\delta T|_{x=\Delta}$ versus the thickness of the plate Δ (in m) with $\lambda_{fw}=10^{-1}$ W/(m·K) $\alpha_v=10^4$ W/(m·K); $j c_p$: 1—5; 2—20; 3—50 W/(m·K).

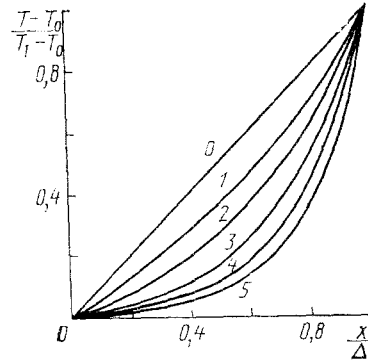


Fig. 3

Fig. 3. The distribution of the temperature within the plate as a function of the coordinate x for different values of the parameter $\psi_1 \Delta = 0; 1; 2; 3; 4; 5$

this is shown in Fig. 2. The temperature is described by the expression

$$T_g(x) \approx T_{fw}(x) \approx T_0 + (T_1 - T_0) \frac{\exp(\psi_1 x) - 1}{\exp(\psi_1 \Delta) - 1}. \quad (18)$$

The distribution of the temperature in the plate is shown in Fig. 3. The efficiency of the thermal shielding in this case is equal to

$$n = \frac{R_r^*}{R_r^0} = \frac{\exp(\psi_1 \Delta) - 1}{\psi_1 \Delta}. \quad (19)$$

Its dependence on the parameter $\psi_1 \Delta$ is shown in Fig. 4.

It should be noted that in the last case the assumption made above, namely $\lambda_{fw} \gg \lambda_g$, is by no means necessary. Moreover, the additive form of the representation of the effective thermal conductivity $\lambda_{eff} = \lambda_{fw} + \lambda_g$ itself is of no significance. The effective thermal conductivity of the material in the atmosphere of filtering gas at rest λ_{eff} should enter in the formula determining ψ_1 instead of λ_{fw} .

In the practical applications of the expressions obtained by solving the system of equations (6) in (6') the term α_v , characterizing the intensity of the thermal interaction between the porous material of the plate and the gas filtering through it, introduces the greatest uncertainty. The quantity α_v , or the coefficient of internal heat transfer, depends on the structure of the porous material, the velocity of the gas flowing through it, and its physical parameters. Its value can be determined, for example, from the criterional equation presented in [5]. In practice it often turns out that the condition (17) holds, thanks to which it is permissible to use the simplified expressions (18) and (19), describing the process of active thermal shielding under conditions of local temperature equilibrium between the framework and the gas.

In summarizing what was said above we note that active thermal shielding gives a many-fold increase in the thermal resistance of porous materials; this is equivalent to the use of hypothetical thermal insulators with an extremely low thermal conductivity.

NOTATION

c_p , mass heat capacity at constant pressure; j , mass flux density; q , heat flux density; h , enthalpy; R_T^0 , R_T^* , thermal resistance of the plate in the regime of passive and active heat shielding, respectively; T , temperature; x , coordinate; α_v , coefficient of heat transfer; Δ , thickness; λ_{eff} , effective coefficient of thermal conductivity of the porous mater-

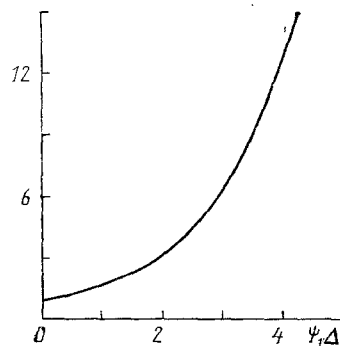


Fig. 4. The efficiency of active thermal shielding versus the parameter $\psi_1\Delta$

ial; λ_{fw} and λ_g , component of the effective thermal conductivity of the porous material determined by its framework and the gas phase, respectively; and, ξ and ψ , roots of the characteristic equations.

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ENERGY DISSIPATION IN A SOUND WAVE IN THE PRESENCE OF EVAPORATION AND CONDENSATION AT A SURFACE

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We consider the transmission and absorption of plane and spherical acoustic waves in the presence of evaporation and condensation at a flat surface. A sound absorbing device is considered as an example.

The reduction of noise is a crucial problem in many fields of technology: ship-building and aircraft construction, architecture, radio, television, and concert studios, and in manufacturing plants. Noise from internal sources is reduced using devices based on absorption of sound waves caused by friction in porous bodies, resonators, surface vibrations, and so forth. Noise from external sources can be reduced by means of sound insulation (sound-proofing), where, together with energy absorption, reflection and refraction of waves on the boundary between media with different impedances are also important. The search for new ways of dissipating sound wave energy is important in both sound absorption and in sound insulation.

Hence it is of interest to consider the interaction of sound waves propagating through a saturated vapor with the surface between two phases. Indeed, pressure oscillations in the gas caused by the incident wave excite velocity oscillations at the interface because of the Hertz-Knudsen condition [1-3] and therefore the intensities of the reflected and refracted waves change. As shown in [4-6], the intensity of the reflected wave can be significantly reduced as a result of evaporation and condensation at the surface. This result is obtained